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# Classical Relativistic Particles

R. ARENS
University of California, Los Angeles, California

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**Abstract.** The concept of Lorentz-invariant classical elementary particle is made precise and it is found that there are exactly two families: (0) the well-known free point-particles distinguished only by their masses and (+) a family with eight-dimensional phase space whose members are distinguished either by their mass or their positive spin.

## 1. Introduction

An invariant elementary one-particle system has a space of states $M$ in which the space-time group (here Poincaré, i.e., inhomogeneous Lorentz group) operates or acts in a transitive way. We require transitivity in our definition because if there were a non-trivial subset of states which was invariant then this would define a more elementary system.

We require also that for each state $m$ (or initial condition for some Lorentz frame) there should be defined a position and velocity $\pi(m)$, to be thought of as the state of the centroid. This map $\pi$ should be such that the centroid moves in the way a free point-particle is always supposed to move.

The last requirement is that there should be a Poisson bracket defined for functions on $M$ and "generating" functions $h_1, \ldots, h_{10}$ such that the infinitesimal space-time transformations $X_1, \ldots, X_{10}$ are represented by $X_i f = \{h_i, f\}$.

In the present paper, we *classify only the cases where M is connected.* The result is that there is allowed not only the free point-particles with various positive masses, but also systems with eight-dimensional state space. These are distinguished by their masses and their positive spin value.

For different values of these parameters the systems are not canonically equivalent. However, those with non-zero spin have general features in common with the example (mass 1 and spin 1, usually called $\frac{1}{2}$) already presented in [3, Section 5]. It was also shown that this system is the correspondence-limit of the Dirac system.

In all systems of spin zero, the space $M$ is essentially $\boldsymbol{R}^6$ (real cartesian six-space) and the action of $\mathscr{P}$ there is the same for all. The individual particle parameter resides in the Poisson bracket. In the systems of non-zero spin, $M$ is essentially $\boldsymbol{R}^6 \times S^2$ where $S^2$ is the set of unit vectors in $\boldsymbol{R}^3$.