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Classification of the Ricci Tensor*

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Abstract. This paper contains a classification of the Ricci tensor $R_{\alpha\beta}$. The method of derivation is analogous to the spinor version of the Petrov classification of the Weyl tensor. It is shown how the various classes are related to the number and type of eigenvectors and eigenvalues of $R_{\alpha\beta}$. The classification is useful in the geometrization of various fields. The case of a real scalar field is treated in detail.

1. Introduction

In a recent paper [1] the "already unified" field theory of Rainich, Misner, and Wheeler, including the null case, was derived using a classification of the Ricci tensor which is somewhat analogous to the spinor version of the Petrov classification [2, 3] of the Weyl tensor. However, the classification was given only as far as necessary for the remainder of the paper. Here we shall develop it in detail. As a further application we shall consider the geometrization of a massless real scalar field. Results obtained previously by Brill [4] and Penney [5] are found to be incomplete.

In Section 2 we shall explain the notation. Section 3 deals with the classification of the Ricci tensor $R_{\alpha\beta}$ and Section 4 with the relation of this classification to the number and type of eigenvectors and eigenvalues of $R_{\alpha\beta}$ [6]. In Section 5 we apply this classification to the problem of geometrization of a massless real scalar field. In the appendix we exhibit canonical forms for the dyad components of the tracefree Ricci tensor for various classes.

2. Notation

The notation used is the same as in Ref. [1]. Tensor indices are given by small Greek letters and run from 1 to 4. Spinor indices are denoted by capital Latin letters and run from 1 to 2. The summation convention is used throughout. Ordinary differentiation is indicated by a comma,

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