Commun. math. Phys. 20, 220—244 (1971)

Irreducible Lie Algebra Extensions of the Poincaré Algebra

II. Extensions with Arbitrary Kernels

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Received July 24, 1970

Abstract. We analyse the extensions of the Poincaré algebra $\mathscr{P}$ with arbitrary kernels. The main tool is a reduction theorem which generalizes the Hochschild-Serre theorem for $n = 2$. This reduction theorem is proved and used to investigate the structure of the Lie algebras obtained by extension.

We look particularly for the irreducible and $\mathscr{R}$-irreducible extensions of $\mathscr{P}$ and we classify the types of irreducible extensions with arbitrary kernels.

Introduction

We pursue here the analysis of the irreducible extensions of the Poincaré algebra begun in [1]. In this II. Part we concentrate on the more complicated problem of extensions with arbitrary kernels.

The difficulties in the non-abelian case have their roots in the fact that the Chevalley-Eilenberg cohomology can not be directly used. As a consequence the set of extensions with fixed character of a given Lie algebra by a non-abelian Lie algebra can also be empty. But something of the Chevalley-Eilenberg cohomology subsists also if the kernel is non-abelian: we have a pseudocohomology (cohomology in Calabi's sense [2]) which allows us to generalize the results in [1]. This pseudocohomology is defined only if $n = 2$, since it is intimately related to the extension theory of Lie algebras. Then we are able to generalize the Hochschild-Serre theorem for $n = 2$ to a reduction theorem valid also in the non-abelian case.

Starting from this result it is possible to develop an extension theory of the Poincaré algebra with arbitrary kernels.

We introduce in Section I the ideas of prerepresentation and pseudocohomology [2]. We show how they are linked to the theory of Lie algebra kernels [2–5].

The preinessential extensions, which form the bridge between the extensions with arbitrary kernels and those with abelian kernels, are considered in Section II.