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Time Development of Quantum Lattice Systems

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Abstract. The time development of quantum lattice systems is studied with a weaker assumption on the growth of the potential than has been considered previously.

I. Introduction

The problem of describing the time development of a statistical mechanical system has not yet been treated satisfactorily. In the algebraic approach to statistical mechanics, it has often been assumed that time-translations correspond to automorphisms of the algebra of quasi-local observables [1]. This assumption has been justified in a few very special cases [2–4], but is not true in general. In particular, it has been shown to be invalid for the ideal Bose gas and BCS models [5]. Indeed, it would be rather surprising if such an assumption were generally valid because it would imply that even those states which are not physically realizable have a well-behaved time development. Therefore, it would seem desirable to study the time-development in a simple case without this assumption. In this paper we consider the time-development of a quantum lattice system. Our assumptions about the growth of the potential are less restrictive than those of Robinson [2], which imply that time-translations correspond to automorphisms of the algebra.

A lattice system is one which is parametrized so that it can be identified with $\boldsymbol{Z}^\nu$, the space of ν-tuples of integers. A Hilbert space, $\mathscr{H}(x)$, of finite dimension, N, is associated with each lattice site x in $\boldsymbol{Z}^\nu$. The Hilbert space

$$\mathscr{H}(\Lambda) = \bigotimes_{x \in \Lambda} \mathscr{H}(x)$$

is associated with each finite region Λ in $\boldsymbol{Z}^\nu$. The algebra of local observables for Λ, $\mathfrak{A}(\Lambda)$, is simply the algebra of bounded operators on $\mathscr{H}(\Lambda)$. If $\Lambda_1 \subset \Lambda_2$, one can identify every A in $\mathfrak{A}(\Lambda_1)$ with the operator $A \otimes I_{\Lambda_2 \setminus \Lambda_1}$ in $\mathfrak{A}(\Lambda_2)$, where $I_{\Lambda_2 \setminus \Lambda_1}$ is the identity on $\mathscr{H}(\Lambda_2 \setminus \Lambda_1)$. Then one

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