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On Representations of the Canonical Commutation Relations*

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Abstract. In the measure space construction of a representation of the canonical commutation relations, the strong continuity of any one parameter subgroup is proved.
All multipliers for the separable case are expressed in a constructive manner and an irreducibility criterion for a subset of multipliers is obtained.

§ 1. Introduction

For a pair of a linear space V_ϕ and a subspace V_π of its algebraic dual V_ϕ^*, a representation of CCR (canonical commutation relations) is unitary operators $U(f)$ and $V(g)$ for each $f \in V_\phi$ and $g \in V_\pi$ satisfying

$$U(f_1)\, U(f_2) = U(f_1 + f_2)\,, \tag{1.1}$$

$$V(g_1)\, V(g_2) = V(g_1 + g_2)\,, \tag{1.2}$$

$$U(f)\, V(g) = V(g)\, U(f)\, e^{-ig(f)}\,. \tag{1.3}$$

It is usually required that $U(\lambda f)$ and $V(\lambda g)$ are strongly continuous in the real parameter λ for each fixed $f \in V_\phi$ and $g \in V_\pi$.

Let μ be a V_π-quasi-invariant probability measure on (V_ϕ^*, B_ϕ), where B_ϕ is the σ-algebra generated by cylinder sets. The standard representation of CCR on $H_\mu = L_2(V_\phi^*, B_\phi, \mu)$ is given by $U_\mu(f)$ and $V_\mu(g)$ defined as follows:

$$[U_\mu(f)\, \Psi]\, (\xi) = e^{i\xi(f)}\, \Psi(\xi)\,, \tag{1.4}$$

$$[V_\mu(g)\, \Psi]\, (\xi) = [d\mu(\xi + g)/d\mu(\xi)]^{1/2}\, \Psi(\xi + g)\,. \tag{1.5}$$

Here $\Psi \in H_\mu$ and $\xi \in V_\phi^*$ [1, 7].

The continuity of $U_\mu(\lambda f)$ in λ is easily proved but the continuity of $V_\mu(\lambda g)$ in λ is not known in the literature for non-separable space (cf. [9, 10]). We shall prove continuity of $V_\mu(\lambda g)$ in λ in Section 2.

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