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# Normal and Locally Normal States

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**Abstract.** It is shown that if $\mathfrak{A}$ is an irreducible $C^*$ algebra on a Hilbert space $\mathscr{H}$ and $N$ is the set of normal states of $\mathfrak{A}$ then the weak and uniform topologies on $N$ coincide and are identical to the weak*-$\mathfrak{A}$ topology for each $\mathfrak{A} \supset \mathfrak{L}\mathfrak{C}(\mathscr{H})$. It is further shown that all weak* topologies coincide with the uniform topology on the set of normal states with finite energy or with finite conditional entropy. A number of continuity properties of the spectra of density matrices, the mean energy, and the conditional entropy are also derived. The extension of these results to locally normal states is indicated and it is established that locally normal factor states are characterized by a doubly uniform clustering property.

## 1. Introduction

We consider subsets of normal and locally normal states over $C^*$ algebras which have the property that all induced weak* topologies coincide. These considerations are motivated by quantum statistical mechanics where the physically relevant structure appears to be given by subsets of locally normal states equipped with the weak* topology of the associated quasi-local algebra. As there is an inherent ambiguity in the choice of the local algebras which generate the quasi-local algebra, it is of interest to find that on most sets of physical relevance, e.g. the set of states with finite conditional entropy, all possible choices of the local algebras lead to the same weak* topology. It is also shown that the continuity properties of a number of interesting functions, such as the energy or conditional entropy, are also independent of the choice of local algebra.

## 2. Normal States and Density Matrices

Let $\mathscr{H}$ be a Hilbert space. A density matrix is defined to be a non-negative, trace-class, operator $\varrho$ on $\mathscr{H}$ normalized such that

$$\mathrm{Tr}_{\mathscr{H}}(\varrho) = 1 .$$