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Constraints Imposed upon a State of a System that Satisfies the K.M.S. Boundary Condition

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Abstract. Using the uniqueness of the K.M.S. automorphism, we investigate the set of automorphisms that commutes with it. The results are applied to gauge invariant quasi-free states of a fermion system.

§ 1. Introduction

The purpose of this note is to investigate some properties of K.M.S. states especially with respect to other possible symmetries.

Let us first recall briefly some definitions and basic features of a K.M.S. state. We refer to [1, 2] for more details.

Definition 1.1. *For a given state ω, the representation $t \to \alpha_t$ of the additive group of real numbers in the $*$-automorphisms group of $\mathscr{A}$ with the property that $\omega(A\alpha_t B)$ is a continuous function of t, is called an evolution.*

Usually such an evolution is a grand-canonical evolution in the sense that it contains the chemical potential. Typically for a system in a finite box V, with Hamiltonian H_V and particle number operator N_V, one has

$$\alpha_t^V(A) = \exp i(H_V - \mu N_V)\, t\, A \exp - i(H_V - \mu N_V)\, t$$

for any bounded operator on Fock space relative to the finite box V.

Definition 1.2. *A state ω_β of a c*-algebra $\mathscr{A}$ is said to be a K.M.S. state with respect to an evolution $t \to \alpha_t$ of $\mathscr{A}$ at the inverse temperature $\beta > 0, \beta < \infty$ if*

$$\int_{-\infty}^{+\infty} f(t - i\beta)\, \omega(A\alpha_t B)\, dt = \int_{-\infty}^{+\infty} f(t)\, \omega(\alpha_t B . A)\, dt$$

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