Commun. math. Phys. 19, 106—118 (1970)

A Classification of Local Current Algebras

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Received May 5, 1970

Abstract. A simple, yet rigorous derivation of all possible forms of a local Lie algebra $\mathscr{L}g$ subject to a certain finiteness condition is presented. This result is used to describe all possible continuous finite dimensional representations of $\mathscr{L}g$.

1. Introduction

The program for the study of the hadrons through current algebra [1–3] presents the problem of finding all possible representations of the so-called local Lie algebras. Here a *local Lie algebra* $\mathscr{L}g$ is defined to be a Lie algebra (over the real or complex numbers) with generators $V_\alpha(\boldsymbol{k})$: $\alpha = 1, 2 \ldots \mu$; $\boldsymbol{k} \in R^n$ (Euclidean n-space), satisfying the commutation relations

$$[V_\alpha(\boldsymbol{k}), V_\beta(\boldsymbol{k}')] = c^\gamma_{\alpha\beta} V_\gamma(\boldsymbol{k}+\boldsymbol{k}') \tag{1.1}$$

where the $c^\gamma_{\alpha\beta}$ denote the structure constants of an arbitrary Lie algebra g. (The summation convention is used for the Greek indices throughout). We shall for the most part assume g to be semisimple and finite dimensional and in what follows we refer to g as the Lie algebra on which the local Lie algebra $\mathscr{L}g$ is based. In physical applications of (1.1), the $V_\alpha(\boldsymbol{k})$ are identified with the currents expressed in the momentum co-ordinates $\boldsymbol{k}$. The Lie algebra g is supposed to represent an approximate symmetry of the strong interactions, which may for example be $su(2)$, $su(3)$ or $su(3) \times su(3)$. The larger local algebra $\mathscr{L}g$, in which g is embedded, mixes this internal symmetry with the external space-time co-ordinates and physically may be thought to play the role of a spectrum generating algebra.

Local Lie algebras belong to the class of infinite dimensional Lie algebras and for these a representation theory has not yet been fully developed. In the meantime, Chang, Dashen, and O'Raifeartaigh [4] have suggested studying local Lie algebras whose generators admit an expansion as a finite sum of the form

$$V_\alpha(\boldsymbol{k}) = \sum_{r=1}^{m} a_r(\boldsymbol{k}) V^r_\alpha \tag{1.2}$$

where the $a_r(\boldsymbol{k})$ are real or complex-valued functions carrying the dependence of the momentum variable $\boldsymbol{k}$. In the following we give a rigorous