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Quantum Stochastic Processes II

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Abstract. We investigate properties of a class of quantum stochastic processes subject to a condition of irreducibility. These processes must be recurrent or transient and an equilibrium state can only exist in the former case. Every finite dimensional process is recurrent and it is possible to establish convergence in time to a unique equilibrium state. We study particularly the class of transition processes, which describe photon emissions of simple quantum mechanical systems in excited states.

§ 1. Introduction

In earlier papers [1–3] we have shown that in order to treat repeated measurements or measurements extended over a period of time, it is necessary to extend considerably the conventional description of measurement theory in quantum mechanics. In order to describe regorously the photon counting experiments being done in quantum optics, for example, it was necessary to develop a theory of quantum stochastic processes [3]. These are generalisations of classical Markov processes and can be analyzed in terms of, and constructed from, two infinitesmal generators. The first of these is the Hamiltonian of the quantum mechanical system, and the second is a stochastic kernel, describing how the measuring instrument interacts with that system. In the presence of the measuring instrument the system evolves according to a one-parameter strongly continuous semigroup of positive endomorphisms of a space of self-adjoint trace class operators.

In this paper we start the analysis of the properties of a class of (quantum stochastic) processes. A process is called irreducible if it cannot be restricted to any proper closed subspace of the underlying Hilbert space and we restrict attention throughout to the irreducible processes. As in classical probability theory the reducible processes are of a much more complex nature and cannot be "decomposed" as direct integrals of irreducible ones. A class of processes, called simple, has the property that certain order ideals associated with compact subsets of the value space of the process are finite-dimensional. We establish necessary and sufficient conditions on the infinitesmal generators of a process for it to be simple and irreducible. We also prove that the simple irreducible

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