

Ultralocal Scalar Field Models

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Abstract. In this paper the quantum theory of ultralocal scalar fields is developed. Such fields are distinguished by the independent temporal development of the field at each spacial point. Although the classical theories fit into the canonical framework, this is not the case for the quantum theories (with the exception of the free field). Explicit operator constructions are given for the field and the Hamiltonian as well as several other operators, and the calculation of the truncated vacuum expectation values is reduced to an associated single degree of freedom calculation. It is shown that construction of the Hamiltonian from the field, as well as the transition from the interacting to the noninteracting theories entails various infinite renormalizations which are made explicit.

1. Introduction

In this paper we consider the quantum theory of scalar field models for which the classical Hamiltonian reads

$$H = \int \left\{ \frac{1}{2} \pi_{c_l}^2(\mathbf{x}) + \frac{1}{2} m_0^2 \varphi_{c_l}^2(\mathbf{x}) + V[\varphi_{c_l}(\mathbf{x})] \right\} d\mathbf{x}.$$

Here $\pi_{c_l}(\mathbf{x})$ and $\varphi_{c_l}(\mathbf{x})$ denote the classical momentum and field, respectively, and \mathbf{x} is a point in configuration space (of arbitrary dimension). Such a theory is *ultralocal* – differing from a relativistic theory by the absence of the term $\frac{1}{2} [\nabla \varphi_{c_l}(\mathbf{x})]^2$ – and it is distinguished by the independent evolution of the dynamics at each point of space¹. This feature, at once the key to their solubility, is likewise responsible for the extremely delicate nature of these models.

The quantum theory of such models does not fit into the canonical framework whenever there is nontrivial interaction due to the lack of canonical commutation relations. There is a field operator $\varphi(\mathbf{x})$ (requiring a spacial smearing function alone), and a Hamiltonian \mathcal{H} , and consequently there exists a time-dependent field

$$\varphi(\mathbf{x}, t) = e^{i\mathcal{H}t} \varphi(\mathbf{x}) e^{-i\mathcal{H}t}.$$

However, there is no $\dot{\varphi}(\mathbf{x}) = i[\mathcal{H}, \varphi(\mathbf{x})]$ operator. The domains of the operators $\varphi(\mathbf{x})$ and \mathcal{H} just do not permit $\dot{\varphi}(\mathbf{x})$ to be defined. The simple

¹ A number of studies of these models have appeared previously [1].