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Constraints on the Derivatives of the $\pi\,\pi$ Scattering Amplitude from Positivity

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Abstract. Conditions derived from positivity are given both above and below threshold for the derivatives of the $\pi\,\pi$ scattering amplitude.

1. Introduction

From the general assumptions of unitarity, crossing and analyticity in a domain derivable from axiomatic field theory, it can be shown that the $\pi\,\pi$ scattering amplitudes satisfy twice subtracted dispersion relations [1]. Hence D and higher partial waves have the Froissart-Gribov representation [2] so that if $f_l(s)$ are the partial waves in the s channel for even isotopic spin states where s is the centre-of-mass energy squared[1],

$$f_l(s)=\frac{4}{\pi(4-s)}\int_4^\infty A_t(s,t)\,Q_l\left(\frac{2t}{4-s}-1\right)dt;\ l=2,4,\ldots \tag{1.1}$$

when $0<s<4$. $A_t(s,t)$ is the absorptive part of the scattering amplitude[2] and for certain isotopic spin combinations[3] in the s channel it has the expansion

$$A_t(s,t)=\sum_{l=0}^{\infty}(2l+1)\,\alpha_l(t)\,P_l\left(1+\frac{2s}{t-4}\right) \tag{1.2}$$

where the $\alpha_l(t)\geqq 0$ from unitarity.

It follows from (1.2) that for $t\geqq 4$ and $0<s<4$,

$$A_t(s,t)\geqq 0\,. \tag{1.3}$$

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[1] We take units such that the mass of the pion is unity.

[2] We will use the suffices s and t to indicate physical quantities in the s channel and t channel respectively.

[3] See Eq. (2.1).