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Superstable Interactions in Classical Statistical Mechanics

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Abstract. We consider classical systems of particles in ν dimensions. For a very large class of pair potentials (superstable lower regular potentials) it is shown that the correlation functions have bounds of the form

$$\varrho(x_1, \ldots, x_n) \leqq \xi^n .$$

Using these and further inequalities one can extend various results obtained by Dobrushin and Minlos [3] for the case of potentials which are non-integrably divergent at the origin. In particular it is shown that the pressure is a continuous function of the density. Infinite system equilibrium states are also defined and studied by analogy with the work of Dobrushin [2a] and of Lanford and Ruelle [11] for lattice gases.

0. Introduction

A number of papers have been devoted to the study of the thermodynamic limit (infinite volume limit) in the statistical mechanics of classical systems of particles in ν dimensions. Fairly satisfactory results have been obtained for the thermodynamic functions: existence of the limit, convexity (stability) properties, and the equivalence of the various ensembles [1]. For other problems (continuity of the pressure as a function of specific volume, study of correlation functions) the results are less satisfactory due to a technical difficulty: it is hard to exclude large fluctuations of the number of particles in a small region of space. It is true that if many particles are put in a small region Δ of space their repulsion will lead to a large positive potential energy (and therefore to a small probability in the grand canonical ensemble), but it is difficult to estimate the interaction energy of the particles in Δ with the neighbouring ones. In the present paper we solve the technical difficulty just mentioned and study some consequences of the solution.

[1] See the pioneering work of Van Hove [15], Yang and Lee [16] and the articles of Ruelle [13], Fisher [5], Griffiths [7]. For a general exposition and further references see Ruelle [14].