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On the Scattering Operator for Quantum Fields*

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Abstract. We study quantum fields interacting by the interactions usually considered in the theory of elementary particles. That is we take the interaction density to be a polynomial P in the fields, and assume that $P = P_b + P_y + P_w$, where P_b is a fourth order polynomial in the boson fields only, P_y is linear in the boson fields and P_w is a polynomial in the fermi fields only. After introducing a space and momentum cut-off in the interaction we prove that the scattering operator exists for all values of the cut-off parameters. We then introduce the scattering operators of relativistic quantum fields as weak limit points of cut-off scattering operators as the cut-off is taken away.

I. Introduction

We consider in this paper a finite number of interacting boson and fermion fields. We will assume that the interaction density is a real polynomial P in the fields themselves, and that P is of the form $P = P_b + P_y + P_w$. P_b is a polynomial in the boson fields only, which is of fourth order and as a polynomial of real variables P_b is bounded below. P_y is a polynomial which is linear in the boson fields and of even degrees in the fermion fields. P_w is a polynomial of the fermion fields only, which is of even degrees in the fermion fields. We shall refer to the three terms in P as the boson self interaction, the Yukawa interaction and the weak interaction respectively.

Let $\phi(x)$ by any of the fields we consider. We then define the momentum cut-off field by

$$\phi_\varepsilon(x) = \int_{\mathrm{R}^3} g_\varepsilon(x-y)\,\phi(y)\,dy \tag{1.1}$$

where $g_\varepsilon(x) \varepsilon\, C_0^\infty(\mathrm{R}^3)$ and converge to the δ-distribution as ε tends to zero. We shall assume that the free energy H_0 defined as a self adjoint operator on the Fock space $\mathscr{F}$ with domain D_0 is such that all the free fields have strictly positive masses. In that case we know that $\phi_\varepsilon(x)$ is, in the boson case a self-adjoint operator with domain containing D_0, and in the fermi case it is a bounded operator on $\mathscr{F}$. The cut-off interaction is now given

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