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Equilibrium States for a Classical Lattice Gas

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Abstract. Various definitions of thermodynamic equilibrium states for a classical lattice gas are given and are proved to be equivalent. In all cases, a set of equations is given, the solutions of which are by definition equilibrium states. Examples are the condition of Lanford and Ruelle, and the KMS boundary condition. In connection with this, it is shown that the time translation for classical interactions exists as an automorphism of the quantum algebra of observables, under conditions which are weaker than those found for quantum interactions.

1. Introduction

In this paper, various definitions of a thermodynamic equilibrium state for a classical lattice gas will be compared.

According to the grand canonical prescription, the equilibrium state for given temperature and chemical potential is found as the limit of finite volume Gibbs states. In some cases however (phase transitions) the limit is not unique. Therefore one may try to define equilibrium states directly for the infinite system. There are two ways known.

1. *The Variational Principle.* An equilibrium state is a translationally invariant state, that maximizes the pressure [5]. By the occurrence of the mean entropy, translations play an essential role.

2. *The KMS-condition and other Equivalent Conditions.* A set of equations is given, solutions of which are by definition equilibrium states.

In the present paper we will concentrate on the second way. We will find, that it works independent of the presence, if any, of lattice translations. Therefore they will not be taken into account.

In Section 4, a number of equivalent equilibrium conditions are given. One of them is a condition by Lanford and Ruelle [7] and Dobrushin [12, 13]. When restricted to invariant states, this condition is equivalent to the variational principle [7].

In Section 6, the KMS-condition is shown to be equivalent to the conditions of Section 4.

As a preliminary step to the KMS-condition, time translations are studied in Section 5. In order to get a non-trivial automorphism, we have to imbed the classical algebra into the quantum algebra (Section 2).