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Lorentz Covariance of the $\lambda(\varphi^4)_2$ Quantum Field Theory

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Abstract. We prove that the $\lambda(\varphi^4)_2$ quantum field theory model is Lorentz covariant, and that the corresponding theory of bounded observables satisfies all the Haag-Kastler axioms. For each Poincaré transformation $\{a, \Lambda\}$ and each bounded region $\boldsymbol{B}$ of Minkowski space we construct a unitary operator U which correctly transforms the field bilinear forms: $U\varphi(x, t)\, U^* = \varphi(\{a, \Lambda\}\,(x, t))$, for $(x, t) \in \boldsymbol{B}$. We also consider the von Neumann algebra $\mathfrak{A}(\boldsymbol{B})$ of local observables, consisting of bounded functions of the field operators $\varphi(f) = \int \varphi(x, t) f(x, t)\, dx\, dt$, $\operatorname{supp} f \subset \boldsymbol{B}$. We define a $*$-isomorphism $\sigma_{\{a,\Lambda\}}: \mathfrak{A}(\boldsymbol{B}) \to \mathfrak{A}(\{a, \Lambda\}\, \boldsymbol{B})$ by setting $\sigma_{\{a,\Lambda\}}(A) = U A U^*$. The mapping $\{a, \Lambda\} \to \sigma_{\{a,\Lambda\}}$ is a representation of the Poincaré group by $*$-automorphisms of the normed algebra $\cup_{\boldsymbol{B}} \mathfrak{A}(\boldsymbol{B})$ of local observables.

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