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A Point Mass in a Einstein Universe

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Abstract. An exact solution of Einstein's equations is presented for a perfect fluid representing a point mass in a Einstein Universe.

Unlike De Sitter's and Friedman's cosmological solutions for which a generalization with a point mass is known to exist [1, 2], such corresponding generalizations was not yet found for the Einstein Solution. This gap is here filled.

Let us consider the line element given by

$$ds^2 = -\left(1 - \frac{2m}{r}\right)^{-1} [1 - a(r-m)^2]^{-1}\, dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) + \left(1 - \frac{2m}{r}\right) dt^2 . \tag{1}$$

The calculated components of the energy-momentum tensors are

$$-8\pi(T_1^1 = T_2^2 = T_3^3) = \frac{-a(r-m)^2}{r^2}, \tag{2}$$

$$8\pi\varrho = \frac{a}{r^2}(3r - 5m)(r - m). \tag{3}$$

It represents therefore an ideal fluid. For $a = 0$ the element reduces to that of the Schwarzschild's exterior solution. For $m = 0$ we have the Einstein universe.

The correct signature is obtained for

$$2m < r \quad \text{and} \quad (r-m)^2 < \frac{1}{a}. \tag{4}$$

The two inequalities are compatible for $m^2 a < 1$. Provided that this last inequality is satisfied it is possible to find a range of values for r so that (4) holds.