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Asymptotic Locality and the Structure of Local Internal Symmetries

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Abstract. Symmetries are investigated from the local viewpoint. Using the Haag-Ruelle construction, the action of a local internal symmetry on the asymptotic states is determined. A condition of "asymptotic locality" is derived and used to show that the symmetry acts linearly and locally on the asymptotic fields. Within a field theoretical framework it is shown that the internal symmetry must commute with the Poincaré group. The general structure of an internal symmetry is determined. The uniqueness of the representation of the Poincaré group is discussed, and a simple example of an infinite component field is given to indicate what occurs when there are infinitely degenerate particle multiplets.

1. Introduction

There has been renewed interest in recent years concerning the structure of symmetries, primarily due to the consideration of groups such as $SU(6)$, which contain particles of different spin within a single multiplet, or groups which might contain mass-splitting, that is, particles of different masses within a single multiplet. The question arises as to whether it is possible to combine the Poincaré group and another symmetry group in a non-trivial way [1].

In this article symmetry groups are considered from the local viewpoint [2, 3]. To each region R of space-time is associated a set of operators $B(R)$ representing the measurements or operations which can be performed with laboratory apparatus confined to the region R. If R_1 and R_2 are space-like separated regions, the condition of locality states that $B(R_1)$ commutes with $B(R_2)$:

$$[B(R_1), B(R_2)] = 0 \quad \text{for} \quad R_2 \subset R_1' \tag{1}$$

where R_1' denotes the region space-like to R_1. In the field theoretical framework locality is expressed by

$$[\phi_\alpha(x), \phi_\mu(y)] = 0 \quad \text{for} \quad (x-y)^2 < 0 \,. \tag{2}$$

In such a local theory we expect a symmetry operation to reflect the local properties (1) or (2). Consider a true symmetry of the physical system, i.e. a symmetry operation represented by a unitary operator G