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Riemann Surfaces and Partial Wave Models

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Abstract. Within the context of simple partial wave models for elastic scattering the problem of uniformizing the partial wave amplitude and classifying its Riemann surface is studied. Starting with the analytic continuation of the amplitude an analysis of the Riemann surface is made through its group of covering transformations relative to a simpler base surface. A model based on the Yukawa potential is studied in this manner and the Riemann surface of interest is found to be the universal covering surface of the thrice punctured sphere. The uniformization of the amplitude can be done explicitly in this case by use of the elliptic modular function. In terms of the uniformizing variable, the original discontinuity relations for the amplitude then reduce to functional equations involving elements of the modular group.

I. Introduction

In high energy physics single variable analyticity is of particular usefulness in the study of partial wave amplitudes for four-point functions where the energy is the only complex variable of interest. Through partial wave dispersion relations and the N/D technique, single variable analyticity gives a method of calculating simple partial wave models which can aid the qualitative understanding of many processes. The simplest such models use two-body elastic unitarity together with some assumed left hand cut from which information one must construct a physically sensible amplitude.

Because the partial wave amplitudes are multivalued analytic functions, the analyticity is most succinctly expressed through the structure of the underlying abstract Riemann surface on which the amplitude function is defined. Indeed, in the case of a partial wave model, we may separate the problem of constructing the amplitude from that of studying its Riemann surface. In this paper we will study a particular partial wave model based on the Yukawa potential showing how the classical results of the theory of Riemann surfaces enable one to perform a detailed analysis of the particular surface underlying the model. In this model we will find that the Riemann surface of the amplitude is the universal covering surface of a sphere with three points removed, and hence that the amplitude is uniformized by means of the elliptic modular function.