

# Extensions of the Taub and NUT Spaces and Extensions of their Tangent Bundles

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**Abstract.** A system of extensions of the Taub space and the NUT space with the topology due to Misner is constructed having the property: for each incomplete geodesic in these space-times, there is one and only one extension from the system into which the geodesic smoothly continues. Next, the notion of hypermanifold is introduced which is a generalization of tangent bundle of a space-time, and an untrivial hypermanifold is constructed that contains the tangent bundles of the Taub and NUT spaces as proper sub-manifolds, and within which almost all geodesics are complete. Locally, the hypermanifolds do not yield anything new, but they provide much broader choice of global properties than any four-dimensional space-time manifold.

## 1. Introduction

A significant feature of Einstein's theory of gravitation is that the local characteristics of a space-time, measurable in a small neighbourhood of a point, are closely related to the properties of the solution as a whole, and that these global, topological properties may be untrivial, in fact very impressive, and sometimes quite complicated. This is of invaluable importance for such a global theory as cosmology is, where the general relativity provides a language even to formulate problems, to say nothing about their solutions.

On the other hand, the choice of topologies as actually implied by the theory in particular solutions is sometimes restricted enough, so that closed time- or light-like lines violating the last rests of causality in physics cannot be avoided [1].

With the progress in mathematical tools, the interest of physicists in this field increases. We mention the papers of Penrose [2], Hawking [3–5], and Geroch [6], where the famous singularity theorems have been stated and proved: if some more or less verifiable conditions are fulfilled, then a kind of singularity of the given space-time is inevitable. These conditions are highly general in that no special space symmetry and no explicit state equation of matter are assumed. The singular space-time is defined as follows: 1. The space-time is not extendable, or, there is no space-time including the original one as its proper sub-manifold. 2. There