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Stable Potentials, II

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Abstract. A family of stable one-dimensional potentials is shown not to be decomposable into the sum of a non-negative function and a function of non-negative type. This settles in the negative a question raised by Ruelle.

§ 1

In his book on Statistical Mechanics [1] Ruelle raised the question whether every (measurable) stable potential on $\boldsymbol{R}^\nu$ can be decomposed into the sum of a continuous function of non-negative type and a (measurable) non-negative function. In our previous paper on this subject [2] we generalized this problem somewhat and gave examples of stable potentials on $\boldsymbol{Z}_{2k+1}$ $(k \geqq 2)$, the group of integers modulo $2k+1$, which are not capable of such a decomposition. The physically relevant case is that concerned with the group $\boldsymbol{R}^\nu$ (for $\nu = 3$). In the present paper we carry the analysis one step closer to this case. In § 2 we give a two parameter family of potentials $\{\varphi_{t,d}\}$ $(0 < t \leqq 1, -\infty < d < \infty)$ with the following property: There is a critical value $d_0 = d_0(t)$ such that $\varphi_{t,d}$ is stable for $d \geqq d_0$ and unstable for $d < d_0$. In § 3 we consider the particular case $t = 1$ and $d_0(1)$, i.e. a potential that in a sense is critically stable, and show that it cannot be decomposed in the manner suggested by Ruelle. In § 4 we list some unsolved problems suggested by this paper.

§ 2

Let $\varphi(x)$ be a real valued even function of the real variable x. With a given φ and any positive integer n, we associate the function

$$\Phi_n = \Phi_n(x_1, x_2, \ldots, x_n) = \sum_{1 \leqq i,j \leqq n} \varphi(x_i - x_j)\,. \qquad (1)$$

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