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A Duality Theorem for the Automorphism Group of a Covariant System

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Abstract. In this paper we examine the covariant representation theory of a covariant system (A, G) introduced by Doplicher, Kastler and Robinson. (A is a C^*-algebra and G is a locally compact group of automorphisms of A.) We define the concept of left tensor product of two covariant representations. Loosely stated, our duality theorem says that G is canonically isomorphic to the set of bounded operator valued maps on the set of covariant representations of the covariant system (A, G) which preserve direct sums, unitary equivalence and left tensor products. We further show that the enveloping von Neumann algebra $\mathscr{A}(A, G)$ of the covariant system (A, G) admits a (not necessarily injective) comultiplication d such that $(\mathscr{A}(A, G), d)$ is a Hopf von Neumann algebra. The intrinsic group of this Hopf von Neumann algebra is canonically isomorphic and (strong operator topology) homeomorphic to G.

§ 1. Introduction

Generally speaking, the mathematical purpose of any representation theory is to study an abstract and perhaps intractable object by examining the collection of structure preserving maps (morphisms) into some simpler, or at least more concrete object. In the case of the unitary representation theory of locally compact groups, the term "concrete" is more appropriate than the term "simpler" as every locally compact group is isomorphic and homeomorphic to a group of unitary operators acting on a Hilbert space, in the strong operator topology (cf. Lemma 2.2 of [2]). A similar phenomena of course shows up in the case of the $*$-representation theory of a C^*-algebra.

If the examination of the representation theory is to serve as an effective tool for obtaining information about the structure of the original abstract object, we must have a representation theory which completely determines the structure of that object. One of the most basic questions one can ask of any representation theory is: Are there enough representations? Initially we would like to know if there are enough representations to distinguish points. Once we have satisfied ourselves as to this minimal requirement of any representation theory,