Commun. math. Phys. 17, 61—74 (1970)

The Enveloping Algebra of a Covariant System

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Received November 5, 1969

Abstract. In an earlier work, Doplicher, Kastler and Robinson have examined a mathematical structure consisting of a pair (A, G), where A is a C^*-algebra and G is a locally compact automorphism group of A. We call such a structure a "covariant system". The enveloping von Neumann algebra $\mathscr{A}(A, G)$ of (A, G) is defined as a $*$-algebra of operator valued functions (called options) on the space of covariant representations of (A, G). The system (A, G) is canonically embedded in, and in fact generates, the von Neumann algebra $\mathscr{A}(A, G)$. Further we show there is a natural one-to-one correspondence between the normal $*$-representations of $\mathscr{A}(A, G)$ and the proper covariant representations of (A, G). The relation of $\mathscr{A}(A, G)$ to the covariance C^*-algebra $C^*(A, G)$ is also examined.

§ 1. Introduction

In an earlier work [2], Sergio Doplicher, Daniel Kastler, and Derek Robinson have examined a mathematical structure consisting of a pair (A, G), where A is a C^*-algebra and G is a locally compact group of automorphisms of A. In this paper we shall refer to such a structure as a "covariant system". (The formal definition is given in the next section.) In their paper, Doplicher, Kastler and Robinson examine other algebraic objects associated with a covariant system. Specifically they define and study a Banach $*$-algebra $\mathfrak{A}_1^G$, (which is the analogue of the L_1-group algebra of a group), and its C^*-completion under the minimal regular norm, which they denote $\bar{\mathfrak{A}}^G$ (and which we shall denote $C^*(A, G)$). This latter algebra is referred to by Georges Zeller-Meier, as the crossed product of A by G, [10]. Covariant systems together with their associated crossed product (or covariance C^*-algebra) have received some study in the mathematical literature. (cf. [8, 9, and 10].) The important correspondence of the covariant representation theory of a covariant system (A, G) and the proper $*$-representation theory of its covariance algebra $\mathscr{A}_1^G$ (and hence of its crossed-product $C^*(A, G)$) is presented in § III of [2].

The purpose of this paper is to define and examine other algebraic objects which may be canonically associated with a covariant system. In particular we define a von Neumann algebra $\mathscr{A}(A, G)$, as an algebra