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Disjointness of the KMS-States of Different Temperatures

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Abstract. Disjointness of (KMS)-states of different temperatures is proved.

Let A be a C^*-algebra with a one parameter automorphism group σ_t. A state φ of A is said to satisfy the *Kubo-Martin-Schwinger (KMS) boundary condition* for $\beta > 0$ if for every pair x, y in A there exists a function $F(z)$ holomorphic in the strip: $0 < \operatorname{Im} z < \beta$ with boundary values:

$$F(t) = \varphi(\sigma_t(x)y) \quad \text{and} \quad F(t + i\beta) = \varphi(y\sigma_t(x)). \tag{1}$$

If we assume the boundedness of the relevant function F on the whole strip: $0 \leqq \operatorname{Im} z \leqq \beta$, the condition (1) implies the σ_t-invariance of φ by Sturm's Theorem, as is shown by Winnink [11].

In quantum thermodynamics, the above β is given by $\beta = 1/kT$, where k is the Boltzmann constant and T is the absolute temperature of the system. Recently, a great deal of progress on the KMS boundary condition has been done by several physicists, for example, [1, 2, 4, 6, 7, and 11].

From the purely mathematical point of view, the author has shown recently in [9] that to every faithful normal state φ of a von Neumann algebra M there corresponds a unique one-parameter automorphism group σ_t^φ of M with respect to which φ satisfies the KMS boundary condition for $\beta = 1$. The proof is based on Tomita's theory [9, 10]. This σ_t^φ is called the *modular automorphism group* of M associated with φ.

Therefore, the following question naturally comes into consideration: *How does the modular automorphism group σ_t^φ depend on a normal faithful state φ? What changes will occur in the modular automorphism group σ_t^φ for different normal faithful states?*

In this paper, we shall show the relation between σ_t^φ and σ_t^ψ for two normal faithful states φ and ψ commuting in the sense of [9: Definition 15.1], that is, when $\varphi + i\psi$ and $\varphi - i\psi$ have the same absolute