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# Continuity Properties of the Representations of the Canonical Commutation Relations* **

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**Abstract.** We prove that a given representation of the canonical commutation relations can be extended uniquely by continuity to larger test function spaces which are maximal in the sense that no further extension is possible. For irreducible tensor product representations of the canonical commutation relations we give a necessary and a sufficient condition for the admissible test functions. We consider the problem of finding topologies on the test function spaces such that this extension can be obtained by a topological completion. Various examples are discussed.

## 1. Introduction

There is a variety of literature on the representations of the canonical commutation relations (hereafter referred to as CCRs) [1–3, 6–12, 14–22]. In the present paper we are concerned with the continuity properties of representations of the CCRs, and with topologies for the test function spaces. There is some discussion of this problem in the literature. Lemma 2.3 of Araki and Woods [2] gives a criterion for the continuity of the operators $U(f, g)$ and provides a method for extending by continuity a given representation to a larger class of test functions. However this lemma was stated somewhat ambiguously. In fact the present paper is essentially a clarification of this lemma. Streit [22] considered irreducible tensor product representations of the CCRs and gave a numerical criterion for a class of admissable test functions. Chaiken [6] constructed some representations of the CCRs which exhibit rather pathological continuity properties. After the present work was completed, the related results of Reid [18, 19] came to our attention. Recently Hegerfeldt and Klauder [12] have discussed the weakest vector topologies on $V_\phi$, $V_\pi$ such that the maps $f \to U(f)$, $g \to V(g)$ are strongly continuous. A detailed discussion of the associated topologies on the test function spaces will be given in a paper with H. Araki [4].

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