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Independence of Local Algebras in Quantum Field Theory

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Abstract. It is shown that local C^*-algebras $\mathfrak{A}(O_1)$ and $\mathfrak{A}(O_2)$ associated with spacelike separated regions O_1 and O_2 in the Minkowski space are independent. The proof is accomplished by a theorem concerning the structure of the C^*-algebra generated by $\mathfrak{A}(O_1)$ and $\mathfrak{A}(O_2)$.

I. Introduction

Let $\mathfrak{A}$, $\mathfrak{A}_1$, $\mathfrak{A}_2$ be C^*-algebras with $\mathfrak{A}_1$ and $\mathfrak{A}_2$ contained in $\mathfrak{A}$. Picking a state φ_1 of $\mathfrak{A}_1$ and a state φ_2 of $\mathfrak{A}_2$ one may ask whether there exists a state φ of $\mathfrak{A}$ whose restriction to $\mathfrak{A}_i$ equals $\varphi_i (i=1,2)$. If this is the case for any choice of the pair φ_1, φ_2 then we shall say that the algebras $\mathfrak{A}_1$ and $\mathfrak{A}_2$ are "statistically independent".

In a Quantum Field Theory let $\mathfrak{A}(O)$ denote the algebra of observables which are associated with the region O of the Minkowski space. We use the symbol $O_1 \times O_2$ to denote that two regions O_1, O_2 lie totally spacelike to each other. In [1] Haag and Kastler raised the question of whether two algebras $\mathfrak{A}(O_1)$ and $\mathfrak{A}(O_2)$ are statistically independent when $O_1 \times O_2$.

If $O_1 + x \times O_2$ for $x \in \mathcal{N}$, $\mathcal{N}$ being a suitably chosen neighbourhood of the origin, we write $O_1 \mathbin{⋇} O_2$. Starting from standard assumptions of Quantum Field Theory, Schlieder [2] derived the following

Proposition (Schlieder). *Suppose $O_1 \mathbin{⋇} O_2$. If $x \in \mathfrak{A}(O_1)$ and $y \in \mathfrak{A}(O_2)$ are non-vanishing elements, then $xy \neq 0$.*

Schlieder also pointed out that the property $xy \neq 0$ for non-vanishing pairs of elements of two commuting algebras $\mathfrak{A}_1, \mathfrak{A}_2$ is a necessary condition for statistical independence. We shall show here that this property is also a sufficient condition. One has

Theorem 1. *Let $\mathfrak{A}, \mathfrak{A}_1, \mathfrak{A}_2$ be C^*-algebras with unit elements and let $\mathfrak{A}_i \subset \mathfrak{A}$.*