On Non-Linear Realizations of the Group $SU(2)$

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Abstract. The non-linear realizations of compact connected Lie groups are considered mainly from the point of view of algebraic topology. In particular, all homogeneous spaces of the group $SU(2)$ are listed, the construction of a few non-linear realizations of $SU(2)$ is given and the orbit structure of linear and non-linear realizations are discussed.

I. Introduction

Recently the method of effective Lagrangians was used to fit the experimental data [1]. The effective Lagrangians have been considered partially invariant under non-linear realizations of some chiral group. Consequently the problem of non-linear realizations of Lie groups has arisen and has begun to be studied by physicists [2]. In contradistinction to them, we deal with the problem globally by means of the theory of homogeneous spaces. In particular, non-linear realizations of the group $SU(2)$ are treated in detail from the point of view of algebraic topology.

First, in Section II, we formulate the problem and define basic notions. Realizations and transitive realizations as well as two concepts of equivalence of realizations are introduced. Any realization of a given group can be written as a union of transitive realizations and therefore, in order to find all realizations of the group, we have to answer the following questions. How many transitive realizations exist for a given group? What is their structure and dimension? How to construct other realizations of the group from the transitive realizations.

Since these questions are, in general, difficult, we restrict ourselves to the group $SU(2)$. However, the method used for constructing its non-linear realizations can be applied to the other compact Lie groups as well. In Section III we find all transitive realizations of $SU(2)$, that is, all homogeneous spaces of $SU(2)$, by listing all subgroups of the group

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