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Discrete Isotropies in a Class of Cosmological Models

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Abstract. It is shown that a certain class of cosmological models admits discrete isotropies. These models are solutions of Einsteins field equations, characterised by: (1) the matter is described as a perfect fluid, and (2) there exists a group of motions simply transitive on three-surfaces orthogonal to the fluid flow vector.

§ 1. Introduction

In a recent paper [1] Ellis and MacCallum examined in detail properties of solutions of Einstein's field equations for a perfect fluid which admit a three-dimensional group of isometries simply transitive on hypersurfaces orthogonal to the fluid flow. Investigations of the properties of observations in these cosmological models [1] show that all measurable relations such as the magnitude-red-shift relation in any direction are invariant under certain reflections in the rest space of an observer moving with the matter. The main purpose of this paper is to show the existence of a discrete isotropy group in nearly all of these models, which induces the reflection symmetries mentioned above.

Discrete isometries of a Riemannian space must be treated differently from continuous ones, because no linearised geometrical object such as a killing vector field exists. In § 2 it is shown that if a space admits a transitive group of isometries all further isotropies can be determined and are connected with automorphisms of the Lie algebra.

This is applied in § 3 to positive definite three-spaces with a simply transitive group of isometries and it turns out that all these spaces admit at least one discrete isotropy. The space sections of the cosmological models under consideration therefore admit discrete isotropies.

The question is now whether the isotropy in the space sections is induced by an isotropy group of the space time. The models are determined by solving a Cauchy problem with data given on one of the space sections. § 4 shows that every isometry of the Cauchy data corresponds to an

[1] A paper on this by the authors of [1] is to appear.