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Convergence of Bogoliubov's Method of Renormalization in Momentum Space

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Abstract. Bogoliubov's method of renormalization is formulated in momentum space. The convergence of the renormalized Feynman integrand is proved by an application of the power counting theorem.

1. Introduction

A general theory of renormalization has been developed by Bogoliubov for arbitrary local and invariant interactions. It was shown by Hepp that the renormalized Feynman integrals constructed according to Bogoliubov's rules converge to well defined distributions when the regularization is removed [1–4].

In a recent paper [5] a different formulation of Bogoliubov's method was used which works in momentum space and does not refer to a regularization. The starting point of this approach is the integrand I_Γ of the unrenormalized Feynman integral

$$J_\Gamma(p_1 \cdots p_r) = \lim_{\varepsilon \to +0} \int dk_1 \cdots dk_m \, I_\Gamma(k_1 \cdots k_m, p_1 \cdots p_r) \tag{1.1}$$

in momentum space. The integrand R_Γ of the finite part of (1.1)

$$F_\Gamma(p_1 \cdots p_r) = \lim_{\varepsilon \to +0} \int dk_1 \cdots dk_m \, R_\Gamma(k_1 \cdots k_m, p_1 \cdots p_r) \tag{1.2}$$

is defined as a rational function of the internal and external momenta by substracting appropriate counter terms from I_Γ. The method is thus an extension of the original work of Dyson and Salam [6–8] [1]. For handling the overlapping divergencies Bogoliubov's combinatorial technique is used which applies to renormalizable as well as non-renormalizable theories.

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[1] For some references of other methods of renormalization see [9—12].