Commun. math. Phys. 15, 173—200 (1969)

Fields, Observables and Gauge Transformations II

SERGIO DOPLICHER
Faculté des Sciences de Marseille-Luminy

RUDOLF HAAG and JOHN E. ROBERTS
II. Institut für Theoretische Physik der Universität, Hamburg

Received July 10, 1969

Abstract. We wish to study the construction of charge-carrying fields given the representation of the observable algebra in the sector of states of zero charge. It is shown that the set of those covariant sectors which can be obtained from the vacuum sector by acting with "localized automorphisms" has the structure of a discrete Abelian group $\mathscr{G}$. An algebra of fields $\mathfrak{F}$ can be defined on the Hilbert space of a representation π of the observable algebra $\mathfrak{A}$ which contains each of the above sectors exactly once. The dual group of $\hat{\mathscr{G}}$ acts as a gauge group on $\mathfrak{F}$ in such a way that $\pi(\mathfrak{A})$ is the gauge invariant part of $\mathfrak{F}$. $\mathfrak{F}$ is made up of Bose and Fermi fields and is determined uniquely by the commutation relations between spacelike separated fields.

I. Introduction

In a previous paper [1] we studied how the various inequivalent irreducible representations (superselection sectors) of the "algebra of observables" $\mathfrak{A}$ which occur in an irreducible representation of the "field algebra" $\mathfrak{F}$ are related to each other. The algebra $\mathfrak{A}$ was defined as the "gauge-invariant" part of $\mathfrak{F}$. Several assumptions were made concerning the action of the gauge group $\mathscr{G}$, the representation of the Poincaré group and the local structure of the theory. Under these assumptions we found that the question of whether the gauge group is Abelian or not reflects itself in an interesting difference in the structure of the set of sectors. In the case of an Abelian gauge group all sectors are obtained from a single one by applying "localized automorphisms" to the observable algebra. For a non-Abelian $\mathscr{G}$ one must instead apply localized isomorphisms of $\mathfrak{A}$ onto subalgebras.

Since the physical content of the theory is determined by the algebraic structure of $\mathfrak{A}$, one may regard $\mathfrak{F}$ and $\mathscr{G}$ from the physical point of view as auxiliary constructs. This leads to the question: if we are only given the representation of $\mathfrak{A}$ in the vacuum sector, can we construct all other sectors and define an $\mathfrak{F}$ and a $\mathscr{G}$ in such a way that the structural assumptions of [1] are satisfied?