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Electromagnetic Mass Shifts in Non-Lagrangian Field Theory

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Abstract. The electromagnetic interactions of hadrons are considered in a model in which both the hadrons and the photons are spinless. The perturbation theory of this model is developed in a way which takes the strong interactions rigorously into account. An expression for the second-order electromagnetic mass shifts of the hadrons is derived. The customary formula expressing these mass shifts as integrals over the Compton scattering amplitude is shown to be inaccurate. The correct expression differs from the old one by a changed integration contour. In favourable cases the two expressions coincide if the integrations over the four components of the photon momentum are carried out in a certain order. This order of integration must in general not be changed.

I. Introduction

Attempts to calculate electromagnetic mass differences of hadrons to lowest order in the fine structure constant often start from the relation (here written for the meson case)

$$\delta M^2 = \frac{i}{2} \frac{e^2}{(2\pi)^4} \int d^4q \, K_{\mu\nu}(q^2) \, M_{\mu\nu}(p, q), \qquad (1.1)$$

where $K_{\mu\nu}$ is the photon propagator and $M_{\mu\nu}$ is the second-order forward scattering amplitude of a photon with momentum q (not necessarily on the mass shell) on a hadron of momentum p (on the mass shell). This formula has been the source of some puzzlement in the last few years, since current algebra arguments seem to indicate that it leads to an infinite π^+-π^0 mass difference [1]. Even though these arguments are by no means compelling (see, e.g., Ref. 2 for pertinent criticism), it is evidently desirable to know to what extent Eq. (1.1) can be trusted.

Equation (1.1) would be an immediate consequence of the Feynman rules if the hadrons had no interactions apart from the electromagnetic ones. However, the strong interactions cannot be neglected in this problem and this makes a creditable derivation of a formula for δM^2 considerably

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