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A Gårding Domain for Quantum Fields

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Abstract. In all representations of the canonical commutation relations, there is a common, invariant domain of essential self-adjointness for quantum fields and conjugate momenta.

1. Introduction

Let $U_k(s)$, $V_k(t)$ be one-parameter continuous unitary groups on a separable Hilbert space X, satisfying the relations:

$$\begin{aligned} U_k(s)\,V_l(t) &= e^{ist\delta_{kl}}\,V_l(t)\,U_k(s)\,, \\ [U_k(s), U_l(t)] &= 0 = [V_k(s), V_l(t)] \end{aligned} \tag{1.1}$$

for all $k, l = 1, 2, \ldots$ and $s, t \in R$. Such a structure is called a representation of the Weyl relations. In this paper we prove the following theorem (whose consequences for Quantum Field Theory are discussed in § 5).

Theorem 1.1. *Let $(\{U_k(s), V_k(t)\}_{k=1}^{\infty}, X)$ be a representation of the Weyl relations ; denote by p_k the generator of $U_k(s)$, by q_k the generator of $V_k(t)$. Then there exists a Banach space, τ, of sequences of real numbers and a domain D, dense in X, such that for all $\{c_k\}_{k=1}^{\infty} \in \tau$,*

1) $\sum_{k=1}^{\infty} c_k q_k$, $\sum_{k=1}^{\infty} c_k p_k$ *are well-defined and essentially self-adjoint on D,*

2) $\sum_{k=1}^{\infty} c_k q_k D \subset D$, $\sum_{k=1}^{\infty} c_k p_k D \subset D$.

3) *If* $\{c_k^n\}_{k=1}^{\infty} \xrightarrow{\tau} \{c_k\}_{k=1}^{\infty}$ *and* $\varphi \in D$, *then*

$$\sum_{k=1}^{\infty} c_k^n q_k \varphi \to \sum_{k=1}^{\infty} c_k q_k \varphi \quad \text{and} \quad \sum_{k=1}^{\infty} c_k^n p_k \varphi \to \sum_{k=1}^{\infty} c_k p_k \varphi\,.$$

We remark that if we were concerned with only a finite number of q_k and p_k, the conclusions of the theorem would follow from well-known work of L. Gårding on representations of Lie groups. For the Fock representation the theorem was proven by J. Cook [2]. In our proof we use heavily the classification of all representations achieved by Gårding and Wightman [4]; it is briefly described in § 2.

The proof of the theorem is contained in § 3 and § 4. In § 3 we construct a dense set of vectors $D_1 \subset X$. The construction is done so that for