Commun. math. Phys. 14, 329—335 (1969)

Global Aspects of the Cauchy Problem in General Relativity

YVONNE CHOQUET-BRUHAT
Département de Mathématiques – Faculté des Sciences, Paris

ROBERT GEROCH*
Department of Mathematics – Birbeck College, London

Abstract. It is shown that, given any set of initial data for Einstein's equations which satisfy the constraint conditions, there exists a development of that data which is maximal in the sense that it is an extension of every other development. These maximal developments form a well-defined class of solutions of Einstein's equations. Any solution of Einstein's equations which has a Cauchy surface may be embedded in exactly one such maximal development.

1. Introduction

It is a well-known (1) property of Einstein's equations that an appropriate set of initial data given on a spacelike 3-surface may be integrated a finite distance into the future, and that the resulting solution is unique, up to isometry, in a neighborhood of the original 3-surface. A priori, it might appear to be possible that, once the solution has been integrated beyond a certain point in some region, the option, previously available, of further evolution in some quite different region has been destroyed[1]. We would thus obtain two distinct developments of the initial data such that neither could be further extended to include the other. Our main result (Theorem 3) asserts that such a circumstance cannot arise: any given set of initial data has a unique "maximal" development. Thus, without loss of generality, one may always deal exclusively with such maximal developments rather than with the initial data itself.

Our proof of the existence of a maximal development does not involve the particular form of the source-free Einstein equations: it is valid for any system of equations the solutions of which define a hyper-

* National Science Foundation Postdoctoral Fellow. Current address: Dept. of Physics, Syracuse University, Syracuse, New York.

[1] Our arguments will involve in an essential way the fact that we are working with equations for which the manifold is not given a priori, but rather is built up as the solution evolves.