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A Particular non-Atomistic Orthomodular Poset

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Abstract. Every atomic orthomodular lattice is atomistic. We show that the corresponding statement for orthomodular posets fails. The result is of interest in the study of the Algebraic Structure of Quantum Mechanics, see [5].

Section 1. Introduction

A partially ordered set (henceforth called a *poset*) P with a smallest element 0 is *atomic* if every non-zero element of P dominates an atom (a minimal non-zero element) of P [7]; it is *atomistic* if (it is atomic and) every element of P is the supremum of the atoms which it dominates [6]. Not all authors agree with this terminology [2]; even worse, some do not explicitly define the term utilized.

Puzzled by [5], page 267, line 36 the present author posed the following question: *Is every atomic orthomodular poset P atomistic?* The answer is negative (although if P were a lattice the answer would be affirmative). Consider two copies of the power set of the integers and identify corresponding finite and co-finite subsets; with the induced order and orthocomplementation this is an atomic orthomodular poset (which is not a lattice). Any element of this poset which corresponds to an infinite and co-infinite subset of the integers has the property that it is *not* the supremum of the atoms it dominates; hence the poset is not atomistic.

However the Ref. [5] suggests that the additional assumption that every element x of P be finite (every chain from 0 to x is finite) or co-finite (x' is finite) may lead to an affirmative answer to the above question. It is the purpose of this paper to prove that this additional assumption does not remove the pathology; in fact, we present an atomic orthomodular poset which satisfies axioms A.1–A.8 of [5] but is not atomistic (in our sense).

Section 2. The Poset

Undefined terms appear in [2] or [3].

Let Z denote the integers. Let Z_1 and Z_2 be disjoint copies of Z; for $i=1,2$ let $\phi_i: Z_i \to Z$ be the natural bijections; let $X = Z_1 \cup Z_2 \cup \{-\infty, \infty\}$ where $-\infty$ and ∞ are any distinct elements not in $Z_1 \cup Z_2$.