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# Nilpotent Locally Convex Lie Algebras and Lie Field Structures*

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**Abstract.** The purpose of this work is to join Lie field structures with certain infinite-dimensional Lie algebras with locally convex topology. These topological Lie algebras allow topological groups which are a generalization of the connected nilpotent Lie groups. We showed the existence of the continuous unitary representations of the gained groups and then we proved the analogue of Gårding theorem. Using this theorem we established the existence of representations of Lie field structures into Lie algebras of skew-symmetric operators on Hilbert spaces.

## Introduction

Infinite-dimensional Lie algebras with topological structures were already the object of the investigation. The semisimple Lie algebras with the topology of the Hilbert space and a particular property of the composition rule were considered by Schue [17, 18] and by Balachandran [1]. Also the infinite-dimensional Filtred Lie algebras with a topological structure were regarded by Veisfeiler [19]. Our aim is to concentrate on the infinite-dimensional Lie algebras with the locally convex topological structures which are the analogue of the nilpotent finite-dimensional Lie algebras and to look for possible groups related with these algebras.

The work is divided into sections with the following content. Definitions of different classes of the locally convex Lie algebras the analogue of which in the case of the finite-dimensional Lie algebras are always the nilpotent Lie algebras are given in Sections 1, 2, 3, and 4. We have shown in Sections 5 and 6 the relation between certain topological groups and the locally convex Lie algebras of Section 3. A construction of the groups is offered in Section 5 and the connection of the unitary representations of the groups and the representations of the related locally convex Lie algebras is considered in Section 6. We proved an analogue of the results of Gårding [8] which is proved for the locally compact groups. In Section 7 we indicated the possible application of the developed theory in some problems of theoretical physics.

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