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On an Infinite-Dimensional Group

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Abstract. We present here an infinite-dimensional Lie algebra, semi-direct product of the Poincaré Lie algebra $\mathscr{P}$ by an infinite-dimensional abelian Lie algebra. It gives rise to Schur-irreducible subgroups of the unitary group of the (separable) Hilbert space, with a discrete mass-spectrum (real positive isolated mass-eigenvalues). Some related mathematical problems are also examined.

I. Introduction

In [1] an example was given of a Poincaré partially-integrable local representation of a fifteen-dimensional Lie algebra, giving rise to a discrete mass-spectrum. Though this example is physical, we cannot get a unitary representation of a Lie *group* (a thing which would have been of technical commodity) out of it, due to the lack of common analytic vectors [2]. In particular, due to the generator q, we are forced in [1] to introduce two domains S_0 and S_π, the former as a dense domain on which all the commutation relations of the 15-dimensional Lie algebra are verified, the latter being the mass-spectrum domain – on which a 14-dimensional subalgebra is integrable, the trouble being caused by q which even does not leave this domain S_π invariant.

As was already hinted in [1], and because of the results of [3], this example is an optimal one (and of physical interest) within the framework of finite-dimensional Lie algebras containing the Poincaré Lie algebra. Of course the necessity of introducing S_0 besides S_π, as well as the fact that we have only *partial* integrability of the representation are caused by the fact that q is not a periodic function. The last suggests [1] to replace multiplication by q by multiplication by $\sin q$ and $\cos q$ (supposing for simplicity $a = 2\pi$ in the notations of [1]). Doing so we get an *infinite-dimensional* Lie algebra with a common dense set of analytic vectors, namely this Lie algebra is integrable to a group representation. This last fact shows us that we have essentially only two types of possibilities of overcoming the negative results of [3]: either we have mass-

[1] This suggestion was made to us by E. P. Wigner in Trieste in June 1968, and was at the origin of this work.