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Cosets and Ferromagnetic Correlation Inequalities

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Abstract. Consideration of subgroups of the group of all subsets of $N = \{1, 2, \ldots, n\}$ with symmetric difference as the operation leads to new inequalities on correlations for a generalized Ising ferromagnet. An upper bound for the rate of change of $\langle \sigma^R \rangle$ with respect to J_S in terms of correlations and a new, brief proof for the monotonicity of $\langle \sigma^R \rangle$ as a function of J_S are given.

§ 1. Introduction

The success of Griffiths [1, 2] in establishing correlation inequalities (the non negativity of correlation and the monotonicity of correlation as a function of interaction) for Ising ferromagnets suggests the problem of getting other, if not all, correlation inequalities [3, Appendix, (1)] for generalized Ising ferromagnets.

In this paper other correlation inequalities are deduced as a consequence of considering a subgroup $\mathscr{G}_0$ of the group $\mathscr{G} = (2^N, \Delta)$ of subsets of $N = \{1, 2, \ldots, n\}$, the set of spin locations, under the operation Δ of symmetric difference. In particular it is shown (in the notation of [3] which is used in the sequel) that

$$\frac{\beta}{2}(1 + \langle \sigma^R \sigma^S \rangle^2 - \langle \sigma^R \rangle^2 - \langle \sigma^S \rangle^2) \geqq \frac{d\langle \sigma^R \rangle}{dJ_S}.$$

A brief proof of the monotonicity of correlation as a function of interaction yields new correlation inequalities as a result of the need for using a sufficiently strong inductive hypothesis.

§ 2. Cosets

Let $\mathscr{G} =_{\text{df}}$ the group $(2^N, \Delta)$. Consider $J : \mathscr{G} \to R$ and $\mathscr{G}_0 \prec \mathscr{G}$ ($\mathscr{G}_0$ a subgroup of $\mathscr{G}$). For all $A \in \mathscr{G}$, let

$$\tilde{J}_{A\mathscr{G}_0} =_{\text{df}} \sum_{B \in A\mathscr{G}_0} J_B$$

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