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Proof of the First Order Phase Transition in the Slater KDP Model

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Abstract. The Slater KDP model defined on d-dimensional tetrahedral lattices is proved to have a phase transition for which the entropy and energy are discontinuous functions at a transition temperature $kT_c = \varepsilon/\ln 2$, independent of dimensionality.

I. Introduction

It has been realized for some time that the Slater KDP model has a first order phase transition at $kT_c = \varepsilon/\ln 2$, where ε is the anisotropy parameter [1–3]. The exact solutions to the two-dimensional case [4] and a particular one-dimensional case [5] support this, but a rigorous proof has not been given for the general case.

The argument of TAKAHASI [2] and others can be used to establish rigorously the "frozen" nature of the model for $T < T_c$. In particular, the following theorem is proved in Section II:

Theorem 1. *The limiting energy and entropy per site are zero for* $T < T_c$, *i.e.*

$$\lim_{N \to \infty} E/N = 0 \quad \textit{and} \quad \lim_{N \to \infty} S/N = 0 \quad \textit{for all} \quad T < T_c .$$

TAKAHASI's argument also leaves little doubt that the model "thaws" at T_c but it appears very difficult to prove this rigorously. Instead, in Section III use is made of the exact high temperature series expansion to prove

Theorem 2.

$$\operatorname*{Lim}_{T \to T_c +} \lim_{N \to \infty} E/N \geqq \varepsilon/2 \quad \textit{and} \quad \lim_{T \to T_c +} \lim_{N \to \infty} S/N \geqq (k/2) \ln 2 .$$

From Theorems 1 and 2 it follows that the energy per site E/N and the entropy per site S/N are discontinuous at $T_c = \varepsilon/k \ln 2$, which is the definition of a first order phase transition.