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Symplectic Approach to the Theory of Quantized Fields. I*

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Abstract. Our aim in this paper the first one of a series concerned with the problem of field quantization starting from the symplectic structure underlying the classical theory, is to build up the variational theory necesary to all further constructions. The basic notions are the *vertical bundle* $\overline{B}$ and the *structure* 1-*form* θ used to define the *generalized infinitesimal contact transformation* which allows us to state and solve the variational problem related to field physics. *Giving a system of modulevalued differential forms of different degree on the vertical bundle which solutions are the stationary cross sections* is the main result in the paper. In this scheme the Euler-Lagrange classical equations are the expressions induced by such a system of differential forms on any cross section of the vertical bundle. This gives us a complete linearization of the Euler-Lagrange equations and, starting from it, a natural globalization of these equations. Finally, the notion of variational problem invariant by a Lie group is defined in this scheme, Noether's theorem related to such invariant problem is formulated and an intrinsic version of the so-called Noether invariants of classical variational calculus is obtained.

Introduction

The study of the symplectic structures that can be associated with classical fields in a natural way becomes more and more fascinating every day, on account of the important application done in recent years of such structures to the problem of field quantization (see, for instance, [6], [7] and [8]). This study is, on the other hand, important by itself, since it makes definitely clear the field dynamics as it was done some years ago with the ordinary analytic dynamics (see, for instance, [1] and [5]).

The first question to deal with is to decide which symplectic structure must be associated with a given classical field. I. Segal has studied that problem in detail in the case of a scalar field on Minkowski's space-time defined by a non-linear hyperbolic differential equation in partial derivatives. Starting from the *manifold of solutions* of the field equation, he

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