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# Fields, Observables and Gauge Transformations I

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**Abstract.** Starting from an algebra of fields $\mathfrak{F}$ and a compact gauge group of the first kind $\mathscr{G}$, the observable algebra $\mathfrak{A}$ is defined as the gauge invariant part of $\mathfrak{F}$. A gauge group of the first kind is shown to be automatically compact if the scattering states are complete and the mass and spin multiplets have finite multiplicity. Under reasonable assumptions about the structure of $\mathfrak{F}$ it is shown that the inequivalent irreducible representations of $\mathfrak{A}$ ("sectors") which occur are in one-to-one correspondence with the inequivalent irreducible representations of $\mathscr{G}$ and that all of them are "strongly locally equivalent". An irreducible representation of $\mathfrak{A}$ satisfies the duality property only if the sector corresponds to a 1-dimensional representation of $\mathscr{G}$. If $\mathscr{G}$ is Abelian the sectors are connected to each other by localized automorphisms.

## I. Introduction

One of the most characteristic assumptions of relativistic Quantum Field Theory is the principle of locality. In its most general form it may be stated as follows:

There is a correspondence between regions in space-time and algebras of observables

$$\mathcal{O} \to \mathfrak{A}(\mathcal{O}) . \qquad (1.1)$$

Here and throughout this paper $\mathcal{O}$ denotes an open and finitely extended region of space-time; $\mathfrak{A}(\mathcal{O})$ is the algebra generated by all the observables which can be measured within $\mathcal{O}$. If $\mathcal{O}_1$ and $\mathcal{O}_2$ are two regions which lie totally spacelike with respect to each other, $\mathfrak{A}(\mathcal{O}_1)$ and $\mathfrak{A}(\mathcal{O}_2)$ are required to commute. The argument for this is based on Einstein's relativistic causality principle which states that no physical influence is possible between the two regions. Hence a measurement in $\mathcal{O}_1$ should not disturb a measurement in $\mathcal{O}_2$.

Historically, in the development of Quantum Field Theory, it has been found necessary (or at least convenient) to introduce unobservable "local" quantities (e.g. "charged fields", Fermi-Dirac fields etc.). Such quantities, being unobservable as a matter of principle, need not commute

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