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A Remark on Asymptotic Completeness of Local Fields

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Abstract. Assuming the existence of an asymptotically complete Wightman field with non-trivial S-matrix, we construct a local field such that the Haag-Ruelle scattering theory applied to this field leads to $\mathfrak{H}_{\text{in}} \neq \mathfrak{H}$ and $\mathfrak{H}_{\text{in}} \neq \mathfrak{H}_{\text{out}}$.

In the framework of local field theory one can define, using the Haag-Ruelle [1] scattering theory, incoming and outgoing states and the corresponding Hilbert spaces $\mathfrak{H}_{\text{in}}$ and $\mathfrak{H}_{\text{out}}$. It is well-known that the axiom of asymptotic completeness ($\mathfrak{H}_{\text{in}} = \mathfrak{H}$) is independent of the other axioms of field theory. In order to have an unitary S-matrix, it is sufficient to require $\mathfrak{H}_{\text{in}} = \mathfrak{H}_{\text{out}}$. Starting from an asymptotically complete Wightman field with non-trivial S-matrix we shall construct a field which does not fulfill this requirement. The construction will show that in our case asymptotic completeness and unitarity of the S-matrix are destroyed by the fact that the functional of truncated vacuum expectation values can be decomposed into a sum of two such (truncated) functionals.

In the following we consider real scalar Wightman fields. We denote the field operator by $A(x)$, the vacuum state by Ω, the representation of the inhomogeneous Lorentz group by $U(a, \Lambda)$ and the Hilbert space by $\mathfrak{H}$.

In addition to the usual postulates of field theory we require [2]:

(I) Let $\sigma(P)$ be the spectrum of the energy momentum operator P. Then $\sigma(P)$ has the form:

$$\sigma(P) = \{p \mid p = 0\} \cup \{p \mid p_0 > 0,\, p^2 = m^2\} \cup \{p \mid p_0 > 0,\, p^2 \geqq 4m^2\};\; m > 0\,.$$

(II) Let $\mathfrak{H}_1$ be defined by $\mathfrak{H}_1 = \{\Phi \mid \Phi \in \mathfrak{H},\, (P^2 - m^2)\,\Phi = 0\}$, and let $U_1(a, \Lambda)$ be the representation of the inhomogeneous Lorentz group in $\mathfrak{H}_1$. Then $U_1(a, \Lambda)$ is an irreducible representation and has spin 0.

(III) Let P_1 be the projection on $\mathfrak{H}_1$. Then the following is true:

$$(A(x)\Omega,\, P_1\, A(y)\Omega) = i\Delta^{(+)}\,(m^2, x - y)\,.$$

With the notation (taken from a paper by Hepp [3])

$$G = \left\{p \mid p_0 < 0,\, |p^2 - m^2| < \frac{m^2}{2}\right\}$$