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A Continuum Analogue of the Lattice Gas

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Abstract. We construct a Hilbert space $\mathscr{H}$, spanned by vectors $|\mathcal{O}\rangle$, where $\mathcal{O}$ is a bounded measurable set in $\mathbb{R}^\nu$ (ν = dimension of space), and interpret $|\mathcal{O}\rangle$ as a state where all points $x \in \mathcal{O}$ are occupied by an incompressible fluid, and $x \notin \mathcal{O}$ unoccupied. $\mathscr{H}$ is generated by applying unitary "filling operators" $U(\mathcal{O})$ to a cyclic vector $|\phi\rangle$, the completely unoccupied state. The operators $U(\mathcal{O})$ generate a commutative c^*-algebra, of which the hermitian elements are interpreted as the observables of the theory.

All the ∞-divisible representations of the symmetric group of order 2 are found. We give a generalization to a theory with any number of particle types.

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A configuration of a lattice gas is defined, in one dimension, by a sequence of noughts and ones. More generally, [1], in ν dimensions, if there are n types of particle, a configuration is given by a map from Z^ν to $(0, 1, \ldots, n)$. In this paper we show how to realize the configuration space of a system describing a gas in a continuum, where it is assumed that each point is either occupied or unoccupied.

1. Continuous Tensor Products

In their general analysis of complete Boolean algebras of factors, Araki and Woods [2] define a continuous tensor product of a family of Hilbert spaces $\mathscr{H}_x$ relative to a vector field Ω_x, to be $\exp \int_{\oplus} (\mathscr{H}_x \ominus \Omega_x)\, dx$, that is, the Fock space over the direct integral of the spaces perpendicular to Ω_x. Their analysis shows that any complete Boolean algebra of factors can be embedded in such a space. Guichardet [3] has given a definition of continuous tensor products of Banach spaces, and a tentative definition of products of Hilbert spaces. There is no obvious reason why the scalar product he defines on the tensor product should be positive-definite in general. But he is able to prove positive definiteness for an example, in which the base-space is compact, by using the Fock space idea. Independently, D. Dubin and the author [4] working from physical considerations, arrived at a model with indefinite metric, indicating that continuous tensor products do not always exist. Further examples