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Lorentzian 4 Dimensional Manifolds with "Local Isotropy"

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Abstract. We define "locally isotropic" spaces, as spaces in which there exists, in the tangent space at each point P, a subgroup $A(P)$ (of dimension at least 1) of the Lorentz group $L_+^\uparrow$, leaving the Riemann tensor and its 2 first covariant derivatives invariant; the subgroups $A(P)$ are assumed to be conjugate in $L_+^\uparrow$. These spaces admit a group of local isometries G. If I_P denotes the subgroup of G leaving P fixed, then $dA(P) = I_P$. All spaces of petrov type D, admitting local isotropy are determined.

1. Introduction

A 4 dimensional Lorentzian manifold V_4 is a differentiable and orientable manifold on which is everywhere defined a regular metric of hyperbolic normal type. One generally assumes also that a coherent time orientation exists; this is equivalent to the existence of a continuous nowhere vanishing time like vector-field; we make this assumption here. In general relativity it is customary to consider local coordinate transformations which are defined by functions of class C^2, piecewise C^4 [1]. We shall need here slightly stronger assumptions: the second derivatives of the Riemann tensor must be continuous at least piecewise.

V_4 is said to admit an isotropy group at the point P if: 1) there exists a locally compact effective transformation group G of isometries of V_4 operating differentiably on V_4.

2) There exists a subgroup I_P of G which leaves the point P fixed. I_P is called the isotropy group at P. A manifold is said to have local isotropy if in each point P it admits an isotropy group I_P; the I_P's are conjugate subgroups of G.

The transformations of I_P induce linear transformations in the tangent space T_P at P. The set of these linear transformations is a subgroup $A^q(P)$ of $L_+^\uparrow$ of dimension $q \geqq 1$; the $A^q(P)$'s are conjugate subgroups of $L_+^\uparrow$.

It has been shown that, in a C^∞ locally isotropic V_4, $A^q(P)$ leaves the Riemann tensor and all its covariant derivatives invariant [2, 3, 4]. Two problems thus arise:

1) to determine all locally isotropic V_4;

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