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Statistical Mechanics of a One-Dimensional Lattice Gas

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Abstract. We study the statistical mechanics of an infinite one-dimensional classical lattice gas. Extending a result of VAN HOVE we show that, for a large class of interactions, such a system has no phase transition. The equilibrium state of the system is represented by a measure which is invariant under the effect of lattice translations. The dynamical system defined by this invariant measure is shown to be a K-system.

1. Introduction and Statement of Results

Let $\mathbb{Z}$ be the set of all integers $\gtrless 0$. We think of the elements of $\mathbb{Z}$ as the sites of a one-dimensional lattice, each site may be occupied by 0 or 1 particle. If n particles are present on the lattice, at positions $i_1 < \cdots < i_n$, we associate to them a "potential energy"

$$U(\{i_1, \ldots, i_n\}) = \sum_{k \geqq 1} \sum_{\{j_1, \ldots, j_k\} \subset \{i_1, \ldots, i_n\}} \Phi^k(j_1, \ldots, j_k) . \tag{1.1}$$

The "k-body potential" Φ^k is a real function of its arguments $j_1 < \cdots < j_k$ and is assumed to be translationally invariant i.e., if $l \in \mathbb{Z}$,

$$\Phi^k(j_1 + l, \ldots, j_k + l) = \Phi^k(j_1, \ldots, j_k) . \tag{1.2}$$

Let $S \subset \mathbb{Z}$ and K^S be the product of one copy of the set $K = \{0, 1\}$ for each point of S; K^S is the space of all configurations of occupied and empty sites in S; K^S is compact for the product of the discrete topologies of the sets $\{0, 1\}$. Let $\mathscr{C}(K^S)$ be the Banach space of real continuous functions on K^S with the uniform norm and $\mathscr{M}(K^S)$ its dual, i.e. the space of real measures on K^S.

If $S \subset T \subset \mathbb{Z}$ we may write

$$K^T = K^S \times K^{T \setminus S} \tag{1.3}$$

and there is a canonical mapping $\alpha_{TS} : \mathscr{C}(K^S) \to \mathscr{C}(K^T)$ such that

$$\alpha_{TS}\, \varphi(x_S, x_{T \setminus S}) = \varphi(x_S) . \tag{1.4}$$

We denote by α^*_{ST} the adjoint of α_{TS}:

$$\alpha^*_{ST}\mu(\varphi) = \mu(\alpha_{TS}\,\varphi) . \tag{1.5}$$