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On the Algebraic Structure of Quantum Mechanics

J. Gunson
Department of Mathematical Physics
University of Birmingham, England

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Abstract. We present a reformulation of the axiomatic basis of quantum mechanics with particular reference to the manner in which the usual algebraic structures arise from certain natural physical requirements. Care is taken to distinguish between features of physical significance and those introduced for mathematical convenience. Our conclusion is that the usual algebraic structures cannot be significantly generalised without conflicting with our current experimental picture of processes occurring at the quantum level.

1. Introduction

The past few years have seen a widespread renewal of interest in the old problem of just why the particular algebraic structure found in quantum mechanics seems to work so well. One question we may ask is: what natural physical requirements can we find to explain the rather remarkable algebraic properties found in the usual quantum mechanical formalisms, in which self-adjoint operators, representing observable quantities, etc., act upon a Hilbert space of states? Alternatively, we may ask whether yet more general formalisms are possible, still being physically sensible? It may be that a definitive answer to these questions is not possible, in view of the difficulty in deciding just what constitutes a "natural" physical requirement.

The deepest results obtained in earlier attacks on the problem were obtained by von Neumann and coworkers [1, 2, 3]. Two main approaches were used. These we may call the "Jordan algebra" and the "propositional calculus" approaches. In both of these, an attempt was made to work from a set of axioms, possessing as much direct physical significance as possible. However, it was found that the usual quantum mechanical formalism was obtained only at the expense of introducing axioms whose physical significance was far from apparent. This is clearly stated by the authors. In the Jordan algebra approach, the point in question is the *distributivity* axiom without which "an algebraic discussion is scarcely possible" [2]. In the other approach, the relevant axiom is that of the modularity of the lattice of propositions which property is "closely related to the existence of an "a priori thermodynamic weight of states" [3].