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Scattering and Bound State Solutions for a Class of Nonlocal Potentials (S-wave)

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Abstract. The s-wave scattering solution is discussed for a class of nonlocal (non-separable) potentials. Existence and uniqueness theorems are given and the analyticity domain in the k-variable (k = wave number in the C.M. system) is determined. Furthermore it is proved that solutions of the bound state problem exist and a discussion of the square-integrable solutions, which can occur for a real positive value of the energy, is given. In this last case the scattering solution also exists but it is not unique. Finally the S-matrix is introduced and it is proved that it is unambigously defined even if the scattering solution is not unique.

1. Introduction

In a previous paper [1] the Born expansion of the scattering solution for a class of nonlocal potentials was considered. The analysis was restricted to the s-wave Schroedinger equation

$$y''(r) + k^2 y(r) = g \int_0^{+\infty} V(r, s)\, y(s)\, ds \tag{1.1}$$

where g is a real quantity and the following assumptions are made on $V(r, s)$:

a) $V(r, s)$ is a real and symmetric function

$$V(r, s) = V^*(r, s) = V(s, r) \tag{1.2}$$

in order to have a time-reversal invariant and hermitian interaction;

b) $V(r, s)$ is a measurable function of both variables, $0 \leqq r < +\infty$, $0 \leqq s < +\infty$, and a real constant $\alpha > 0$ exists such that:

$$C = \int_0^{+\infty} e^{\alpha r}\, dr \int_0^{+\infty} s e^{\alpha s} |V(r, s)|\, ds < +\infty\,. \tag{1.3}$$