Commun. math. Phys. 6, 29—48 (1967)

Tempered Distributions in Infinitely Many Dimensions

III. Linear Transformations of Field Operators

P. Kristensen
Institute of Physics

L. Mejlbo and E. Thue Poulsen
Institute of Mathematics
Aarhus University

Received April 15, 1967

Abstract. In the present paper we continue investigating spaces of tempered distributions in infinitely many dimensions. In particular, we prove that those linear homogeneous transformations of the canonical pair of field operators, which preserve the commutation relations, can be implemented by an essentially unique intertwining operator. The dependence of this operator on the transformation is studied.

1. Introduction

Summary of results

In two previous papers [6] and [7] (in the sequel quoted as I and II) we have studied certain spaces of tempered distributions in infinitely many dimensions, in particular the space $\tilde{\mathfrak{S}}$, which is essentially identical with the space Σ considered by Borchers [1].

In the present work we investigate linear homogeneous transformations of the canonical pair of field operators; in particular linear transformations induced by the real symplectic group Σ over Schwartz's space $\mathscr{S}$[1]. This group we define as the family of all matrices

$$\mathbf{U} = \begin{pmatrix} U & V \\ \overline{V} & \overline{U} \end{pmatrix}$$

with matrix elements in $L(\mathscr{S}, \mathscr{S}) \cap L(\mathscr{S}^*, \mathscr{S}^*)$ and satisfying

$$\mathbf{U}^* \mathbf{J}\, \mathbf{U} = \mathbf{U}\, \mathbf{J}\, \mathbf{U}^* = \mathbf{J}\,, \tag{1}$$

where

$$\mathbf{J} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The linear transformation induced by $\mathbf{U}$ is then defined as the mapping

$$a(\overline{\varphi}) \rightharpoonup a_{\mathbf{U}}(\overline{\varphi}) \equiv a(\overline{U}^* \,\overline{\varphi}) + a^*(\overline{V}^* \,\overline{\varphi})$$
$$a^*(\varphi) \rightharpoonup a^*_{\mathbf{U}}(\varphi) \equiv a^*(U^* \varphi) + a(V^* \varphi)\,.$$

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[1] In the study of spaces of type $\mathfrak{S}$ in I and II we assumed $\mathscr{S} = \mathscr{S}(\mathbf{R}^1)$. In case $\mathscr{S} = \mathscr{S}(\mathbf{R}^n)$, $n > 1$, spaces of type $\mathfrak{S}$ should be modified in the obvious way.