Spectra of Generators in Irreducible Unitary Representations of Non-Compact Semi-Simple Groups

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Abstract.. Several theorems concerning the spectra of elements of the complexi-
fied Lie algebra in unitary representations of non-compact semi-simple groups are
proved. The principal theorem gives purely Lie algebraic sufficient conditions for
the type of spectrum (point or continuous) of any element of the real Lie algebra.
For elements of special "self adjoint" Cartan subalgebras these conditions are
rephrased in terms of the basis-dependent information most readily available to the
physicist, namely, their hermiticity properties and the values of the structure
constants: roots, etc.

1. Introduction

It should be useful in physics to have spectral criteria for the gener-
ators of non-compact groups represented unitarily in Hilbert space
phrased entirely in terms of their Lie algebraic properties, e.g., in terms
of the Killing form norm. Since one also frequently considers the com-
plexified generator algebra (one must, for example, in using a Weyl
"canonical" basis $\{h_i, e_\alpha\}$ for semi-simple groups), one would like to be
able to develop spectral criteria for at least some, if not all, elements z
of the complexified algebra phrased in terms of the most available (basis-
dependent) information, namely, the values of the roots, the hermiticity
properties of z, etc.

In this paper we prove several theorems about spectra of this type.
They fall short of the ideal in that only sufficient conditions can be
proved. But without a doubt they can be sharpened to "if and only if"'s
if higher Lie algebra invariants beyond the Killing form are introduced.
On the other hand it seems remarkable that such "Lie-algebraic"
spectral theorems can be proved in general, that is, without any reference
to the particular unitary representation. Indeed, this is known to be
impossible for the spectral properties of the enveloping algebra.

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