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Quantum Theory in de-Sitter Space*

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Abstract. A general method for constructing fields in spaces with transitive group of transformations is presented. Quantum-theory of free fields with spin 0, 1/2, and the connection of spin and statistics in de-Sitter space of constant positive curvature are discussed.

1. Introduction

Usually Quantum-field-theory is formulated in Minkowski-space. To the group of motions, the Poincare-group, correspond the fundamental conservation laws of energy-momentum and angular momentum. We study quantum-theory in de-Sitter space of positive curvature $(4+1)$, whose group of motions has 10 parameters as the Poincare-group. Classical fields with spin $\neq 0$, especially spin 1/2 have been discussed by Dirac [1] and later for inst. by Lee and Gürsey [2]. For their approach it is essential to embed de-Sitter space in a flat 5-dim. space, moreover for spin 1/2 the accidental existence of a 4-dim. representation of the de-Sitter group is used. The method of covariant derivatives gives equations for arbitrary spin, but as pointed out in [2], it remains unclear under what representation of the de-Sitter group these equations shall be invariant.

We propose a general method for constructing fields in spaces with transitive transformation group, using a field theoretic version of the so called "induced representations". We apply this formalism to quantum theory of free fields with spin 0, 1/2, restmass $m > 0$, in de-Sitter space. The connection of spin and statistics in de-Sitter space is discussed following W. Pauli [3] for the flat space.

We have proved the cited theorems rigorously. For brevity the proofs are only sketched in an appendix.

Notation $*$ conj. complex.
$+$ hermitian conj.
T transposed.

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