

Large Groups of Automorphisms of C^* -Algebras

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Abstract. Groups of $*$ -automorphisms of C^* -algebras and their invariant states are studied. We assume the groups satisfy a certain largeness condition and then obtain results which contain many of those known for asymptotically abelian C^* -algebras and for inner automorphisms and traces of C^* -algebras. Our key result is the construction in certain “finite” cases, where the automorphisms are spatial, of an invariant linear map of the C^* -algebra onto the fixed point algebra carrying with it most of the relevant information.

1. Introduction

In the works of DOPLICHER, KASTLER, ROBINSON, and RUELLE [7, 13, 15] there is developed a theory for C^* -algebras acted upon by representations of the translation group R^n as $*$ -automorphisms. It is assumed that the C^* -algebra in question is asymptotically abelian, which means roughly that the group of automorphisms obtained, is very large, or somewhat more precisely, that if given two operators in the algebra then large translations of one of them will commute with the other. Recently parts of this theory have been generalized to arbitrary groups by LANFORD and RUELLE [14] in a paper which clarifies the underlying mathematical structure of this theory in some respects, but which leaves other aspects of it open. We shall therefore in the present paper develop a theory for representations of groups as “large” groups of automorphisms of C^* -algebras, thus obtaining a better understanding of the underlying mathematics. It will be shown that our situation is less general than that studied by LANFORD and RUELLE. However, having more structure, we shall be able to obtain stronger results, see Theorems 3.1 and 3.7. In particular, in section 5 we shall recover more of the results known for C^* -algebras asymptotically abelian with respect to R^n . Technically we shall say a group G is represented as a large group of automorphisms of a C^* -algebra \mathfrak{A} if there is a representation $g \rightarrow \tau_g$ of G as $*$ -automorphisms of \mathfrak{A} such that for A self-adjoint in \mathfrak{A}

$$\text{conv}(\pi_\rho(\tau_g(A)) : g \in G) \cap \pi_\rho(\mathfrak{A})' \neq \emptyset$$

for all G -invariant states $\rho = \omega_{x_\rho} \circ \pi_\rho$ of \mathfrak{A} , π_ρ being the canonical cyclic representation of \mathfrak{A} induced by ρ . In addition to the asymptotically abelian