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Collision Cross Sections in Terms of Local Observables*

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Abstract. Asymptotic relations for matrix elements of quasilocal operators are given which generalize and extend the Lehmann-Symanzik-Zimmermann relations. These relations allow the simulation of a coincidence arrangement of particle detectors in the mathematical frame of the theory and thereby the expression of collision cross sections in terms of expectation values of observables.

I. Introduction

Within the framework of Quantum Field Theory particle collisions have always been treated by means of formulas and algorithms which are based on the asymptotic relation I below. Denoting the vacuum state by $|o\rangle$, the state of a single particle of type i and momentum $\mathbf{k}$ by $|\mathbf{k}, i\rangle$[1] with the normalization

$$\langle \mathbf{k}', j | \mathbf{k}, i\rangle = \delta_{ij}\delta^3(\mathbf{k}' - \mathbf{k}) \tag{1}$$

we have the

Asymptotic relation I:

If Q is an arbitrary quasilocal[2] operator with $\langle o| Q |o\rangle = o$ and if the point x moves to infinity in a time-like direction[3] then, for $x_0 \to +\infty$

$$Q(x) \to \sum_i \int d^3k (\langle \mathbf{k}, i| Q(x) |o\rangle a_i^{\dagger\,\mathrm{out}}(\mathbf{k}) + \langle o| Q(x) |\mathbf{k}, i\rangle a_i^{\mathrm{out}}(\mathbf{k})) \tag{2}$$

and for $x_0 \to -\infty$

$$Q(x) \to \sum_i \int d^3k (\langle k, i| Q(x) |o\rangle a_i^{\dagger\,\mathrm{in}}(\mathbf{k}) + \langle o| Q(x) |\mathbf{k}, i\rangle a_i^{\mathrm{in}}(\mathbf{k}))\,. \tag{3}$$

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[1] If the particle has spin we shall, for simplicity, consider here the description of the spin orientation included in the index i.

[2] For a definition of "quasilocal" see the beginning of section II.

[3] We use small Latin letters to denote 4-vectors, boldface letters for 3-vectors. Thus $x = (\mathbf{x}, x_0)$ denotes a point in space-time with the time component x_0 and space components $\mathbf{x}$. The energy-momentum 4-vector of a particle of type i is written correspondingly as $k = (\mathbf{k}, k_0)$ where, of course, $k_0 = (\mathbf{k}^2 + m_i^2)^{1/2}$ and m_i is the particle mass. For the Lorentz scalar product we write $kx = \mathbf{kx} - k_0x_0$.